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Dissuasive queues in the time dependent traffic assignment problem

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Abstract

In this study, variables of arc travel-time functions and traffic performance indicators for real networks are related in order to delve into the validity and representativeness of these variables on flow propagation models. A methodology for estimating performance indicators, for the analysis of travel patterns in time-dependent networks, is suggested. Also, an analysis of data from a congested network with daily travelers is presented. Finally, a queue length variable is incorporated in the time-dependent traffic assignment problem; and a suitable travel-time function from the traffic flow theory that can be applied to arcs of signalized and un-signalized arteries, is introduced.

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Keywords: Traffic performance models, time dependent traffic assignment, link travel times function, mobile queue, rhythm travel

1. Introduction

The traffic assignment models are a great resource to estimate travel patterns in a transportation system, discover performance indicators, and define mobility and accessibility variables. Performance indicators are used to identify emergent properties and distinguish problems and externalities that need to be solved. Traffic management actions supported on information technology and intelligent transportation systems are very important for traveler's route choice and could have a deterrent effect.

Time-dependent traffic assignment models (TDTA) and dynamic traffic assignment models (DTA) include variables which can be used to calculate impacts on pollution, delays and fuel consumption, on urban networks.

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These models, used with intelligent transportation systems, can also be a guide to regulate traffic operative and tactical decisions. The difference between TDTA and DTA models is the resolution of the time interval.

A user equilibrium traffic assignment (UEA) model uses flow-dependent arc time functions (ATF) to obtain travel patterns on congested networks, assuming that travel itineraries last one time interval. This model estimates flow and allows to identify congested arcs, but it does not consider the role of queues on route choice. On the contrary, a TDTA model can give detailed information about traffic progress, propagation of densification waves, queues generation, etc. A traffic problem can be analyzed according to the specificity of performance indicators, which are classified as follows:

- Microscopic indicators, which focus on individual vehicle.
- Mesoscopic indicators, which consider the queue process and turning flow in intersections.
- Macroscopic indicators, which are based on travel patterns and flow propagation estimation in a time period.

Although research on modeling and algorithms for TDTA problems has been developed for three decades, there is a gap on: analytical treatments of flow dynamics, the relationship between the dynamic variables, and the flow behavior in an individual arc belonging to a route chosen by travelers and its spread through the network.

Several studies and papers extensively discuss the DTA problem, its modeling, solution and testing on networks, with both analytical and simulation approaches. However, research must go deeper into this subject, especially on matter of time-dependent travel functions which describe flow propagation on networks.

Researchers agree on the properties of ATF if the network is loaded with static user equilibrium, because the results closely represent the user behavior. On the contrary, in time-dependent traffic assignment modeling, functional forms of time-dependent variables have yet to be studied as well as their properties and relationship with the flow-dependent arc travel time functions (time dependent arc travel time function, TDATF).

Road systems include roads and intersections, topologically expressed like arcs and nodes, where trips are daily loaded. Most of them are regular trips, with the same origin-destination (O-D) pair everyday on paths chosen according to the user's experience. Everyday users know the alternative routes behavior. Travelers could be persuaded to change their routes, if they have information about congestion, bottlenecks and queues. In fact, free open access traffic applications (APP) are becoming widespread (i.e. several application, based on Google Maps, give real time traffic information to users).

Through this study, the TDTA problem is studied by means of analytical and macroscopic approaches. Performance indicators for time-dependent networks are defined and calculated in order to examine and assess the resolution and accuracy of ATF so they are adequate to DTA. This is a current and in progress topic.

We provide a framework for analyzing performance of congested networks (with daily travelers), by means the use of a GPS app and traffic analysis based on the Manual of Transportation Engineering Studies (Schroeder, Findley, Hummer, & Foyle, 2010), and displaying it by using free-access resources. We propose a methodology for obtaining, processing and estimating performance indicators, which contributes to DTA modelling.

This paper has three parts. The first one presents the description, characteristic parameters and variables of dynamic arc time function (DATF) models for DTA. The second part explains the methodology applied to calculate and show traffic behavior on congested time-dependent networks; here, a case study is presented. In the last part, a discussion about the need of extending the TDATF for DTA is included.

2. Dynamic network loading models for DTA

Nomenclature(Nie & Zhang, 2005[a])

$\mathbf{u}(t)$ Entrance flow rate on arc at time t

$\mathbf{v}(t)$ Exit flow rate on arc at time t

$\mathbf{x}(t)$ Flow rate on arc at time t or number of vehicles on arc at time t

$\tau(t)$ Travel time on arc at time t

Γ Parameters vector that reflects arc characteristics (such as travel-time)

$\mathbf{\Omega}(t)$ Variables vector on arc, status updated at time t

λ Number of vehicles lined up on exit node

α_p Free flow travel-time on arc

β_p Arc capacity

$\lambda(t+\alpha_p)/\beta_p$ Queue delay

A DATF is an analytical expression which represents the relationship between dynamic variables of a DTA model: entrance flow $u(t)$, circulating flow on arc $x(t)$, and exit flow $v(t)$, at time *t.* Several DTA formulations hold these three variables, for obtaining dynamic routes. The analytical expression of DATF in DTA models is the basis to define the solution algorithm as well as to guarantee a good model performance and consistent realistic results.

Peeta and Zhou (2006) indicate that the analytical-functional forms that reproduce real time traffic behavior are inexistent because of the complications of real-time traffic dynamics.

Experts consent on DATF properties: flow propagation and travel patterns in time subintervals must be equal to equilibrium assignment on the complete time interval, as on a static network (Carey & Ge, 2007). Other properties are the following: *FIFO* (first-in-first-out), meaning first vehicles to enter, exit faster, and *causality*, which means that speed and travel-time of a vehicle on an arc, are affected by the preceding vehicles (Carey & Ge, 2007). Then, the DATF of arcs must include congestion (UEA criteria), and FIFO and causality rules, in order to obtain solutions according to reality in DTA models (Long, Gao, & Szeto, 2011).

An analysis of strengths and weaknesses of DATF in DTA by Mun (2007) emphasizes its properties and proposes new formulations to surpass some problems. He also points out that the best functions, such as a lineal travel-time one, overcome travel time and do not represent reality. It is necessary to study the properties of dynamic variable functional forms, and their relationship with travel-time arc functions (dependent on the number of vehicles in *t*).

Nie & Zang (2005a) describe a DTA model based on a dynamic arc model known as General macroscopic arc model, with the following constraints:

Flow conservation: $\dot{x}(t) = u(t) - v(t)$ (1)

Flow performance function: $\tau(t) = f(\Gamma, \Omega(t))$ or $v(t) = g(\Gamma, \Omega(t))$ (2)

Flow propagation: $\int_t^{t+\tau(t)} v(s)ds = x(t)$ (3)

Dynamic traffic load (DTL) is a sub-process of DTA problems, as it can provide travel-time and additional data (entrance flow, exit flow, queue length, etc). The approaches based on arc formulations are described as follows.

2.1. Whole-Link Travel Time Models

Whole-Link Travel Time Models (WLTM) (named by Carey & McCarney, 2002) or delay function models (named by Nie & Zhang, 2005b), both mentioned in Ban, Pang, Liu & Ma (2012) calculate traffic propagation using travel-time arc functions dependent on $u(t)$, $v(t)$ y $x(t)$ in a loading interval *t*. Travel-time functions are linear and nonlinear; the linear functions satisfy the desirable DATF mathematical properties in DTA (Nie & Zhang, 2005a), but are unrealistic.

2.2. Point-queue model

In a point-queue model (PQM), flow moves freely on the arc all the way through the exit node. As for the congestion state, it is represented as a vertical line where vehicles reach the head of the arc when the exit rate is bigger than its capacity; then the queue length is zero in each loaded time interval. Nie & Zhang (2005a) present a PQM discretization process (quoted by Ban, Pang, Liu & Ma, 2012) from the following continuous version (Nie & Zhang, 2005a):

$$\frac{d\lambda(t)}{dt} = \begin{cases} 0 & si \quad \lambda(t) = 0 \quad y \quad u(t-\alpha_p) < \beta_p \\ u(t-\alpha_p) - \beta_p & \text{otherwise} \end{cases} \tag{4}$$

$$v(t) = \begin{cases} u(t-\alpha_p) & si \quad \lambda(t) = 0 \quad y \quad u(t-\alpha_p) < \beta_p \\ \beta_p & \text{otherwise} \end{cases} \tag{5}$$

$$\tau(t) = \alpha_p + \lambda(t+\alpha_p)/\beta_p \tag{6}$$

Some authors have proposed methods for solving the mentioned models, WLTM and PQM, which can be set in two groups: flow continuous models and link transmission model.

2.3. Flow continuous models

Flow continuous models are based on LWR theory (Lighthill & Whitham, 1955, & Richards, 1956) and Cell Transmission Model (CTM) (Daganzo 1994, 1995), the discrete version. The LWR model is based on kinematic wave theory, in which traffic is treated as a hydraulic fluid and is represented by the fundamental diagram that relates link flow and density. This model divides the arc into homogeneous sections where traffic flows and spreads from cell to cell with a propagation rule. The size of the sections may be the equivalent to the distance traveled by vehicles moving freely in a time interval. Traffic conditions are updated at successive times. García, Lopez, Niño & Verastegui (2006) describe this model as a traffic representation by differential equations, where the traffic behavior at a space-time point is only affected by the state of the system in a neighborhood of such point. For further information refer to Ziliaskopoulos (2000); Kuwahara and Akamatsu (2001); Ukkusuri (2004); Lo (1999); and Lo & Szeto (2002), among others.

2.4. Link Transmission model (Yperman, 2007, Gentile, 2008, 2010)

Link Transmission model (LTM) is used in macroscopic simulation of DTA, where vehicle's flow is continuous. The flow propagation on arcs has a pattern consistent to kinematic wave theory (LWR) but is represented on the triangular shaped fundamental diagram that has only two wave speeds (Newell, 1993, quoted by Gentile, 2010). Its analytical formulation describes expansion waves on a smoothed fundamental diagram. Also, the model includes process of queuing theory based on traffic theory principles for intersection with traffic lights and other controls on crossings, by the known "node model" to define transition and partition flow. Additionally, LTM indirectly uses point queue model to estimate intersection's delay (Yperman, 2007). The principle is similar to that used by Akamatsu (1997, 2001), which proposes the usage of increasing cumulative output flow functions. This approach is studied due to its potential on DTA; for example this approach is studied by Bliemer, Brederode, Wismans & Smits (2012), Long Gao & Szeto (2011), Gentile (2010), and others.

All these models must be validated and applied for real cases. Hence, we propose a methodology to this end, and we apply it to the dynamic behavior of a real network by obtaining performance indicators, and computing and analyzing them, which represents an advance in real cases.

3. Information model and calculation of performance indicators on time-dependent congested networks

The assignment process is based on the user route choice, which is done in a rational way; the user chooses the lower cost route between an origin-destination pair. Travel-time, distance, cost, congestion and queues are some influential factors in his/her decision. Traffic indicators and location of bottlenecks and queues can be obtained for different days and year periods, by means of travel-time and delay studies (Schroeder, Findley, Hummer, & Foyle, 2010).

A real network performance can be analyzed by means the use of travel rhythm maps and the Chrome Developer Tools (DevTools). Additionally, with the Maximun-Car Technique (Schroeder, Findley, Hummer, & Foyle, 2010), a GPS app for smartphones and MyTracks app, tracks data can be acquired. This way we can obtain .kml and .cvs files. The Application Programming Interface, API (JavaScript Google Maps) third version, allows us to introduce geo-spatial information and appreciate it according to the user’s convenience. The heat maps (Kinney, 1991) show the movement quality in the travel rhythm pointer which represents inverse speed in [min/km]. Table 1 exhibits a matrix code where each color indicate a range of values. We also review .cvs files to get time, speed and other geo-referenced coordinates. In addition, the heat map provides a spatiotemporal view of a path to obtain a dynamic image. And, we can identify detailed information such as:

- The sections that provide efficient mobility.
- The sections with delay set up in records under 10 km/h (standard value).
- The sites and sections with queues.
- Traffic jams, congestion and affected sites.

Travel time, speed and delay graphics are resources for the analysis of space and time variables, and allow to prove DTA applications. Fig. 1 shows the method used to capture traffic behavior on time-dependent congested networks, by the described techniques. Test vehicles method allows to get data for feeding a software or an APP, which display real time travel rhythm maps (on arterial roads).

Access to traffic software for smartphones (APPs) is now a common practice. An interesting proposal for monitoring road areas is available in Astarita, Vaiana, Iuele, Caruso, Giofrè & De Masi (2014).

4. Case study

This case study is located in the south area of Coyoacan municipality (Mexico City), shown in Fig. 2. We consider two routes for an O-D pair. The network has very few options because the origin (east side) and destination (west side) points are too separate from each other, and the routes must cross a bridge on an arterial road (Viaducto Tlalpan). The alternative routes are the following:

- Route 1: Taxqueña-Quevedo-C. del Agua
- Route 2: Taxqueña-Quevedo-D. del Norte-Pinos-Monserrat-Eje 10 Sur

Table 1. Travel rhythm matrix code

Speed (km/h)	Movement quality Rhythm (min/km)	Speed (km/h)	Movement quality Rhythm (min/km)
0 - 10	6.00	> 25 - 30	2.00
> 10 - 15	4.00	> 30 - 40	1.50
> 15 - 20	3.00	> 40 - 50	1.20
> 20 - 25	2.40	> 50	1.00

Both routes are comparable because they present similarities: 7 km length, two roadways separated by a median, six lanes, a 0.2 km traffic light control and speed control devices for some tracks in scholar zones. Ordinary users drive through the routes, congested during rush hours (up to three hours) and present equal travel times, which is representative of the first principle of Wardrop's (1952) traffic equilibrium. Both routes offer same travel time ever.

The purpose is to calculate traffic indicators and identify bottlenecks and queues, in different terms of a year, during morning rush hours, in order to obtain traffic behavior patterns during regular days and compare the results.

We now present a comparative summary of six vehicle data (in table 2), in order to analyze the variations of alternatives route over time. We include some specific events:

- Variations during business days, summer school vacations and specific events such as first day of school in Ciudad Universitaria (CU).
- Mobility quality in terms of rhythm indicator for two comparable alternatives.
- Location of bottleneck, different affectations, and queues length.
- Delays caused by traffic light speed control.

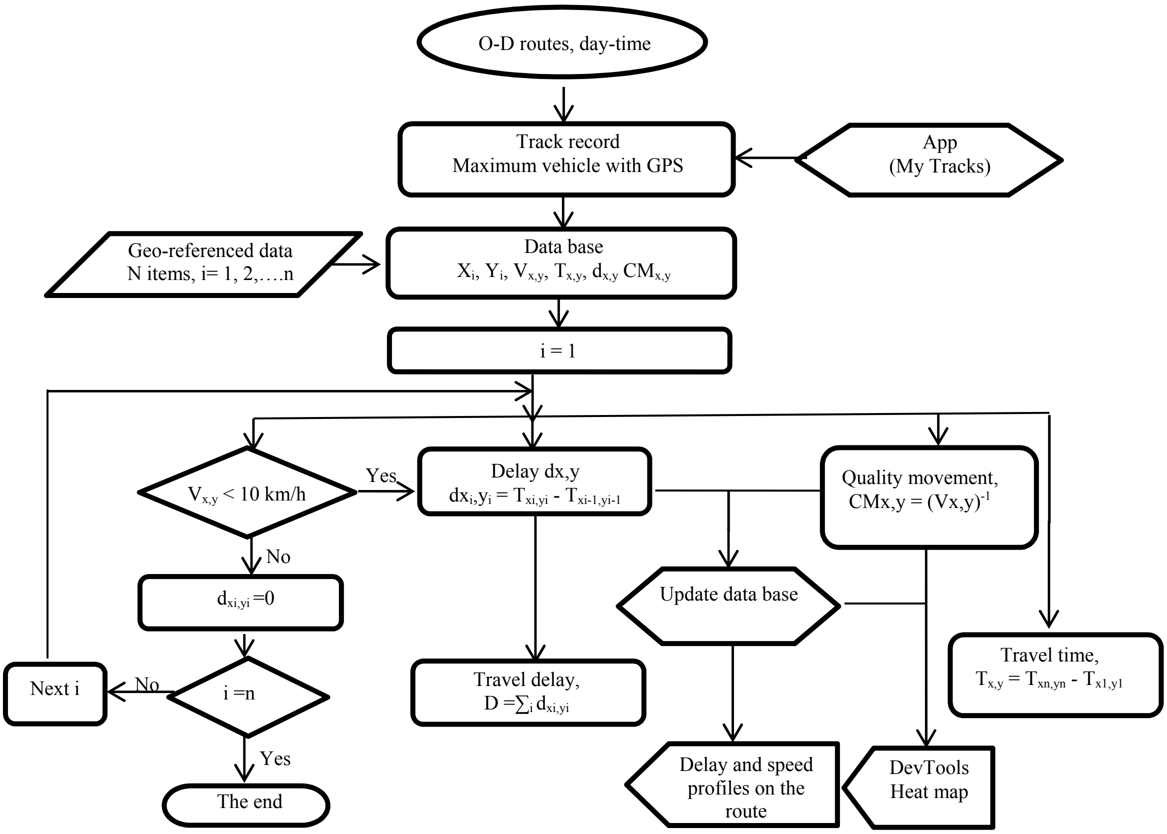

Fig 1. Algorithm for obtaining performance indicators for time-dependent traffic assignment models.

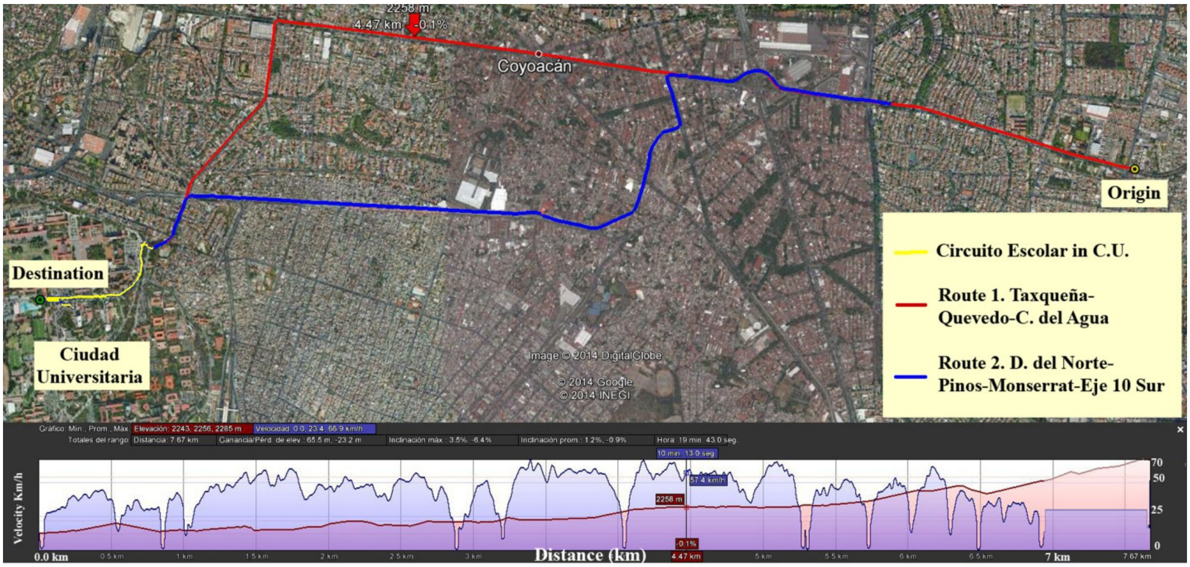

Fig 2. Routes from Origin - Taxqueña and Eje 2 East – Destination: Engineering Tower, Ciudad Universitaria (above). Travel speeds plotted versus distance of Route 1 (down). Source: Original information on Google Earth

Fig. 3 shows mobility details by travel rhythm indicator on "C. del Agua" access towards CU for several periods during years. It also shows the queue influence on temporal-spatial dimensions. This variable is relevant in the route choice and should be part of the travel-time function in traffic assignment algorithms on static and dynamic scope.

We could present several analyses; however, our objective is show the potential of the graphic display device with the travel rhythm indicator calculated by the procedure described in Fig 1. It is interesting the relationship between time and queue size captured by the heat map app. The map records the queue size in delay sites according hour of the day (delay intensity).

We highlight some distinctive findings of the case study:

- Mobility in morning rush hours near "C. del Agua" access causes serious delays, high congestion and high risk for accidents.
- Drivers, students and teachers get frustrated and tired, jeopardizing their academic performance.
- Gas particles and pollution levels increase as well as fuel consumption.
- Walkers and bikers are forced to take long paths due to the congestion and lineups, not to mention the noise and the pollution compromising their health. Also, they are in disadvantage due to risk of accidents.

The equitable indicators of length, travel-time and travel-speed in routes 1 and 2, are summarized in table 2, as well as delays and travel time proportion in delay to emphasize the differences. Also, the variation of the rate indicator in space and time becomes a sign of effectiveness of the road system in the studied area. Note the similarity of the performance indicators in each pair of comparable routes; the only thing that would make the difference in route choice would be the travel rhythm indicator.

TDTA and TDATF models and travel route choice are dependent on: arc vehicular volume, lineups blocking the path, flow propagation on the network and arc control prevailing condition.

Traffic theory uses travel-time functions, suitable for networks with traffic light control on arterial and freeways, accepted to be realistic. It is necessary to integrate such models into TDTA models (Londono & Lozano, 2012).

User's equilibrium assignment model for route choice on congested networks is based on the Wardrop's first principle, where there is a perfect competition market and users have perfect information. Then a traveler cannot improve his/her itinerary for a unilateral substitution of route. If such market is optimized through the time variable, it is modeled as flow dependent. We consider that the model must also include the queue size variable, so the user equilibrium assignment would be modeled as an optimization problem restricted to the travel demand and with supply limited by the feasible queue size, which is time dependent.

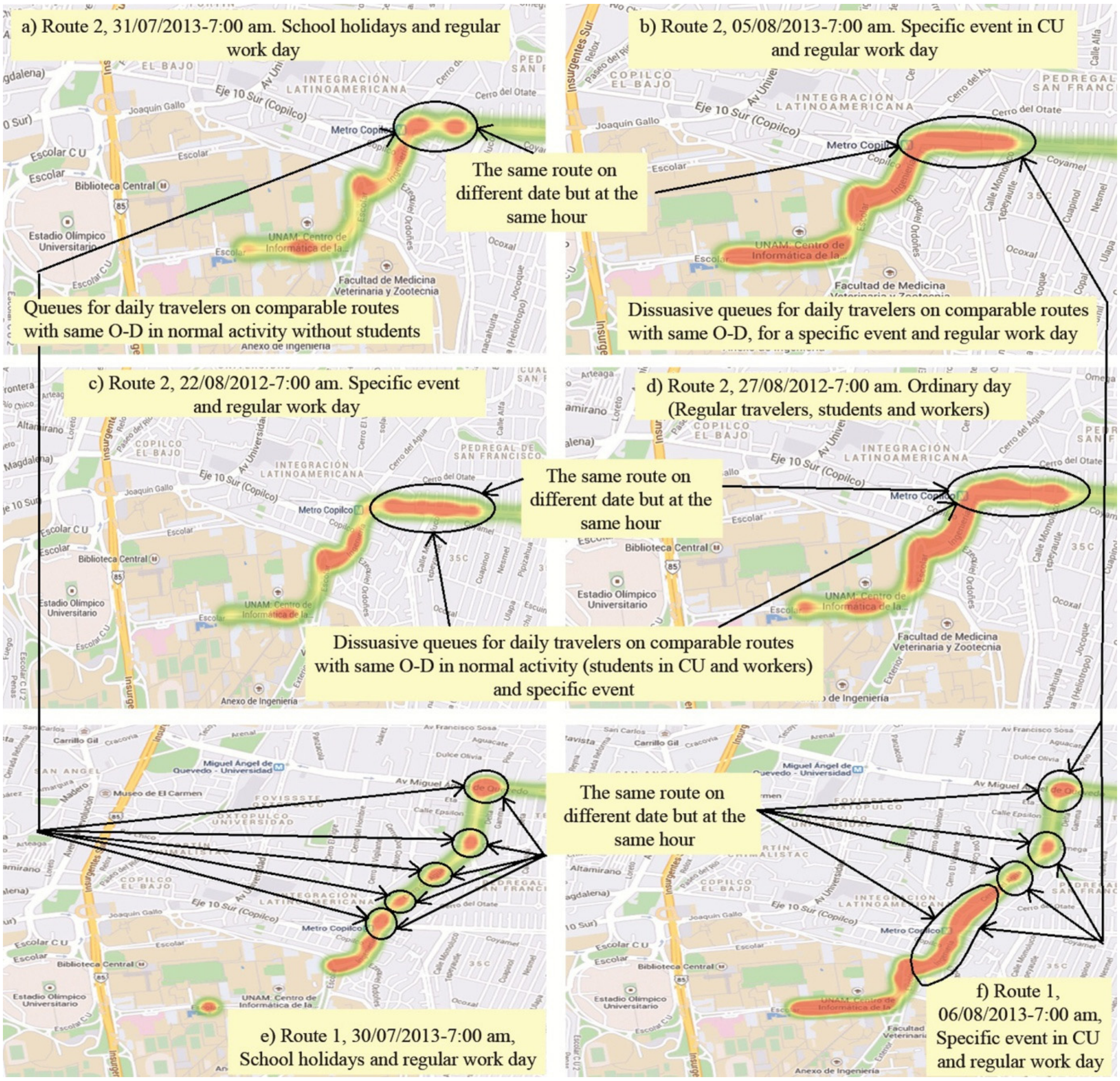

Fig 3. Detailed formation of queues with temporal rhythm indicator (min/km) on several tracks from Taxqueña with Eje 2 Oriente to Ciudad Universitaria (CU), access through C. del Agua (original information with DevTools of Google Chrome, API on Google Maps)

Table 2. Performance indicators for comparable routes

Route	Activity period	Length (*km*)	Mean travel time *(min)*	Mean speed *(km/h)*	Delay (v<10 km/h) *(min/veh)*	Travel time proportion in delay *(%)*	Time	Date	Prevailing traffic condition
1	School vacation and regular labor days	7.68	19.72	22.89	7.77	39.39	6:35:00 am	30/07/ 2013	Low
2	School vacation and regular labor days	7.84	18.12	25.50	4.97	27.41	6:36:00 am	31/07/ 2013	Low
2	First day of school and regular labor days	6.43	37.12	10.39	20.87	56.22	6:38 am	22/08/ 2012	Congestion
2	Ordinary day (school and labor)	6.26	32.37	11.60	17.28	53.40	6:35 am	27/08/ 2012	Congestion
2	First day of school in CU	6.53	44.45	8.72	31.98	71.95	6:39 am	05/08/ 2013	Low but congested access to CU
1	First day of school in CU	6.25	45.15	8.86	29.22	69.32	6:35 am	06/08/ 2013	Low but congested access to CU

We propose a time dependent traffic assignment model through a network design optimization problem using Akcelik's (2003) function, applicable to congested networks. This function depends on the link travel time overflow, the flow-capacity proportion and the initial queue. We also formulate TDAT as a mathematical problem for constrained networks similar to a time-dependent design network problem, but this will be developed in future studies.

5. Conclusion

Macroscopic analysis of time-dependent urban road systems requires powerful, flexible and clear information resources, mathematical models and computational process, in order to predict the performance of the system. However, despite the extensive literature on theoretical and practical studies, there is still required research on macroscopic analysis of urban networks controlled by freeways and signalized arterial access, where flow propagation presents particularities.

We developed a practical and simple method to capture and calculate proper mobility indicators that support research on DTA models and allow assessing the travel patterns performance and traffic propagation on time-dependent networks. Such indicators guarantee reliability on the results.

The case study results are analyzed for going further into time dependent link travel time function variables, suitable to dynamic travel time models used on dynamic network loading. We find a dissuasive impact of queue's length when a traveler is informed. The travel rhythm indicator depends on flow and speed on the arcs, and in a macroscopic analysis it can dimension the size queue. Thus, we propose to include the queue size in the time dependent travel time function for estimating overflow delay. The travel rhythm indicator is a spatial-temporal effectiveness measure for congestion on urban networks.

The time dependent traffic assignment models guide the network management for reactive and proactive making decisions. Currently, traffic operators have technologies for reporting network's prevailing condition, and can dissuade travelers and decrease shockwaves and queuing obstructing roads on important crossings. Some users have smartphones and tablets and APPs which give information on traffic, but it is needed to improve the quality of such information and provide reliable information for route choice.

Traffic performance indicators and the heat maps are a versatile and economical information; they can be obtained by free access applications, useful for analysts and controllers in cities with limited budget.

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